On Tracking Portfolios with Certainty Equivalents on a generalization of Markowitz model: the Fool, the Wise and the Adaptive

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**General investment framework**

- Investor: strategies to find \( \alpha \), to rank portfolios (\( \alpha' > \alpha \) ?)
- General investment framework
  - **Strategy II** - Normative

**Strategy I : (Non-trivial but) oversimplistic**

\[ \alpha \geq \alpha' \Rightarrow E_{\text{norm}}[w \cdot \alpha'] \geq E_{\text{norm}}[w \cdot \alpha] \]

**Strategy II - Normative**

If Make five assumptions about the way the investor computes \( \alpha \geq \alpha' \) :

1. 
2. 
3. 
4. 
5.

For some utility function \( U \).

**Risk premium and Certainty equivalent**

- Theorem:
  - Risk premium and Certainty equivalent associated to \( \alpha \), \( \theta \):
  
**From information geometry pops-up duality allocations vs returns**

- \( \theta \) may be interpreted as a natural market allocation, informative signal for investors (generalizes a result by Markowitz, 1952, for Gaussian: \( \alpha_{\text{unc}} \propto \lambda_j \))

**General learning framework: on-line with three players**

- **General experimental framework**
  - \#4 markets with daily (DJIA, NYSE, TSE) or weekly (S&P500) returns:
  - \#Players (main runs in the supplementary material): \( \alpha \in \{0.01, 1, 100\}, \eta \in \{0.01, 1, 100\}, \psi \in \{M, KL, IS\}, \theta \)

**Experiments**

- **Contenders**:
  - UCRP (Uniform Cost Rebalanced Portfolio) & Best stock
  - OMD (min), OMD (median), OMD (max)

**Properties of OMD**

- **Theorem**: lowerbound on certainty equivalents for OMD
  - \( \text{Fix } \Delta = K/\min(t_1)(\alpha_0 - \alpha_1)^2 \) : then, for any \( \eta > 0 \),
  
**Algorithm: OMD**

- **Lemma**: OMD uses a generalization of Amari’s natural gradient
  - The solution is \( \alpha' = -\eta t \min(t_2)(\alpha_0 - \alpha_1) \)
  - satisfies:
  
**Betting strategies of OMD**

- \( \alpha = 100.0, \eta = 0.01 \) :

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