Approximate null-controllability for parabolic variational inequality

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We are interested in the study of the approximate null-controllability of a parabolic variational inequality with an obstacle denoted ψ:

$$\left\{\begin{array}{rcl} y & \in & \mathcal{K}_\psi, \\ \int_Q \frac{\partial y}{\partial t}(z-y)dxdt + \int_0^T a(y,z-y)dt & \geq & \int_{Q_\omega} v(z-y)dxdt, \ \forall z \in \mathcal{V}, z \leq \psi \\ y(0) & = & y_0. \end{array}\right. \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded open whose boundary $\Gamma = \partial\Omega$ has C^2 regularity, ω is a nonempty open subset of Ω, $Q =]0,T[\times\Omega$, $Q_\omega =]0,T[\times\omega$, $\mathcal{V} = L^2(0,T;H_0^1(\Omega))$, $\mathcal{K}_\psi = \{y \in \mathcal{V};\ \dfrac{\partial y}{\partial t} \in \mathcal{V}',\ y \leq \psi\}$ is a closed nonempty convex and a is a continuous bilinear application on $H_0^1(\Omega) \times H_0^1(\Omega)$ defined by $a(y,z) = \int_\Omega \sum_{i,j=1}^N a_{i,j} \frac{\partial y}{\partial x_i}\frac{\partial z}{\partial x_i} dx + \int_\Omega a_0 yz dx$, with $a_{i,j} \in C^2(\overline{\Omega})$, $a_0 \in L^\infty(Q)$ satisfying elliptic conditons. After introducing a penalized term associated to the obstacle (cf [3]), we show that (1) can be formulated like a weak limit of a sequence of heat semi-linear problems. The approximate controllability theory is applied to this sequence, and by passing to the limit we prove that (1) satisfies approximate null-controllability condition in the sense that there exists an admissible control $\hat{v} \in L^2(Q_\omega)$ such that the corresponding solution $\hat{y} = y(\hat{v})$ of (1) satisfies $\|\hat{y}(T)\|_{L^2(\Omega)} \leq \alpha$. To obtain our main theorem, we have combined some results on approximate controllability obtained by various authors (for instance [4], [1], [5]) for Laplace operators with results based on Carleman inequalities and obtained by Imanuvilov [2]for a class of problems involving second order differential operators with coefficients depending on spatial variables.

References

[1] **C.Fabre, J.P Puel, E. Zuazua**"*Approximate Controllability of the Semilinear Heat Equation*". Proc. Royal Soc, Edinburgh, Vol. 125 A, (1995), pp.31-61.

[2] **O. Yu. Imanuvilov** *"Controllability of parabolic equations"*. Sbornik: Mathematics, Vol. 186, n°6, (1995), pp.879-900.

[3] **J.L. Lions***"Sur Quelques Questions d'Analyse, de Mécanique et de Contrôle Optimal"*. Les Presses de l'Université de Montréal.

[4] **J.P Puel***"Contrôlabilité Exacte et Approchée pour des Systèmes Distribués"*. Note de Cours de D.E.A (2000/2001).

[5] **E. Zuazua***"Controllability of Partial Differential Equations and its Semi-Discrete Approximations"*. Discrete and Continuous Dynamical Systems. Vol. 8, n°2, (2002), pp. 469-513.